An Overview of Adversarial Risk Analysis with applications to Auctions

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(with D. Banks (Duke) and J. Rios (IBM Research))

14 IBIT, Madrid June 2013

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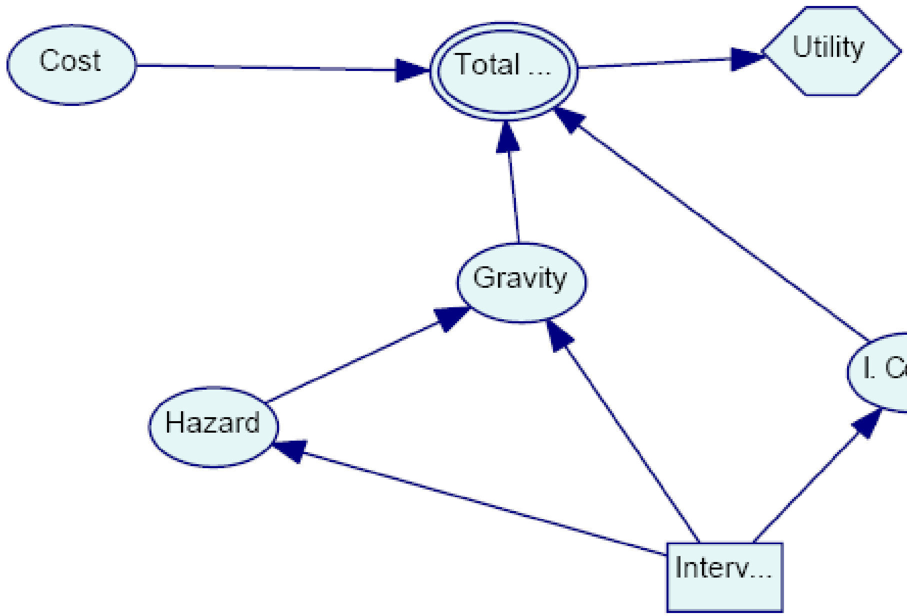
Agenda

- **Adversarial Risk Analysis: Intro**
- Simultaneous Games
- Applications to Auctions
- Conclusions

Risk management

Intervention to be chosen:



Interventions tend to reduce the likelihood of hazard appearance and its gravity… but they also entail a cost

Gain through managed risk

Choose the intervention which provides the biggest gain, if it is sufficiently big…

Which is the best security resource allocation in a city?

City as a map with cells
Each cell has a value
For each cell, a predictive model of delictive acts
Allocate security resources (constraints)
For each cell predict the impact of resource allocation
Optimal resource allocation
Role of insurance

NB: The bad guys operate intelligent and organisedly!!!

FP7 SECONOMICS (Metro Barcelona, UK Grid, Anadolu Airport)

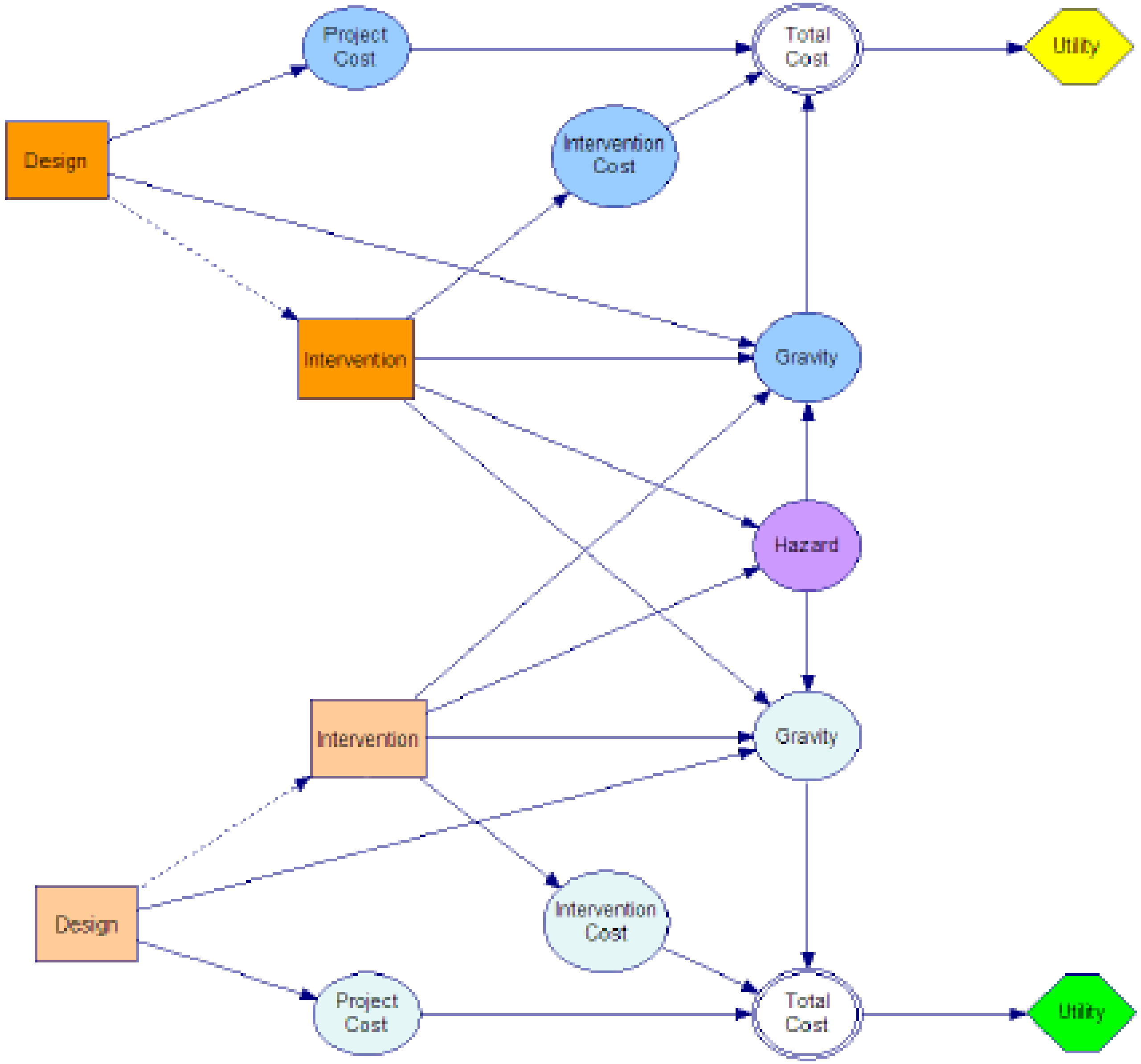
Which is the best HW/SW maintenance for the company ERP?

Model HW/SW system (interacting HW and SW blocks)
Forecast block reliability
Forecast system reliability
Design maintenance policies
Forecast impact on reliability (and costs)
Optimal maintenance policy
Cyberinsurance

NB: What happens with bad guys attacking our system?

RIESGOS (MICINN), RIESGOS-CM (CM)

Adversarial risk analysis



Motivation

- Traditional RA extended to include adversaries ready to increase our risks

- S-11, M-11 lead to large security investments globally, some of them criticised
- Many modelling efforts to efficiently allocate such resources
- Parnell et al (2008) NAS review
  - Standard reliability/risk approaches not take into acocunt intentionality
  - Game theoretic approaches. Common knowledge assumption…
  - Decision analytic approaches. Forecasting the adversary action…
- Merrick, Parnell (2011) review approaches commenting favourably on Adversarial Risk Analysis

- Key business sectors
- Regulatory legislation

Adversarial Risk Analysis

- A framework to manage risks from actions of intelligent adversaries (DRI, Rios, Banks, JASA 2009)

- One-sided prescriptive support
  - Use a SEU model
  - Treat the adversary's decision as uncertainties

- Method to predict adversary's actions
  - We assume the adversary is a *expected utility maximizer*
    - Model his decision problem
    - Assess his probabilities and utilities
    - Find his action of maximum expected utility
  - But other *descriptive* models are possible

- Uncertainty in the Attacker's decision stems from
  - *our* uncertainty about his probabilities and utilities
  - but this leads to a hierarchy of nested decision problems

(random, noninformative, k-level, heuristic, mirroring argument, mixed) vs (common knowledge)

Adversarial Risk Analysis

- ARA applications to counterterrorism models (Rios, DRI, 2009, 2012 Risk Analysis)
  - Sequential Defend-Attack
  - Simultaneous Defend-Attack
  - Sequential Defend-Attack-Defend
  - Sequential Defend-Attack with private information

- Somali pirates case (Sevillano, Rios, DRI, 2012 Decision Analysis)
- Routing games (anti IED war) (Wang, Banks, 2011)

- Borel games (Banks, Petralia, Wang, 2011)

- Kadane, Larkey (1982), Raiffa (1982), Lippman, McCardle (2012)
- Stahl and Wilson (1994,1995)   D. Wolpert (2012)
- Rotschild, MacLay, Guikema (2012)

- Social robotics
- Adversarial signal processing
- Auctions
- …

Adversarial risk analysis

u_1 p_1	u_2 p_2
$\hat{u}_2$ $\hat{p}_2$	

G.T. (Full and common knowledge)

Asymmetric prescriptive/descriptive approach

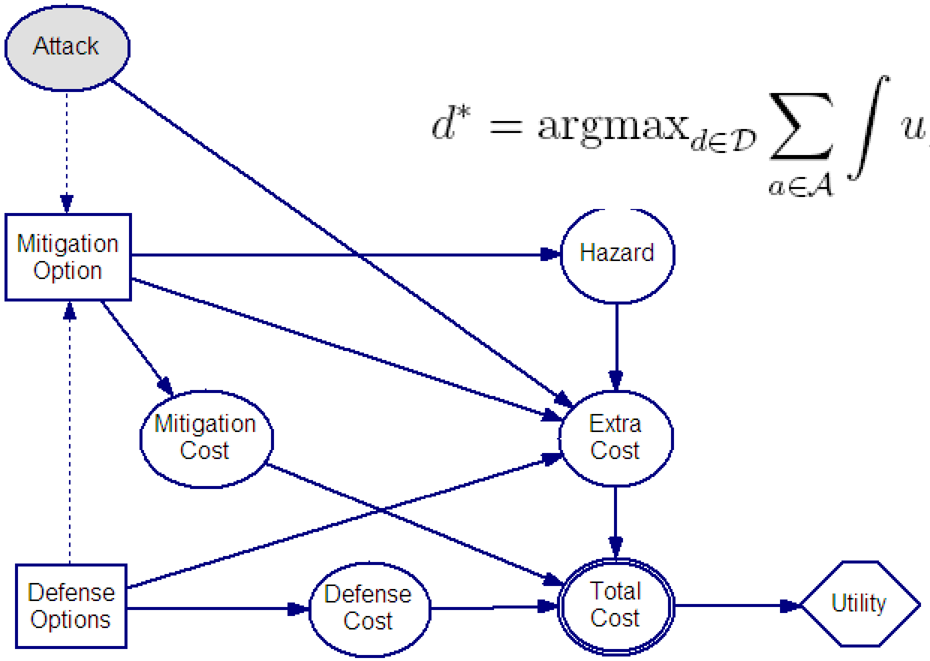
u_1 p_1	u_2 p_2
$\hat{u}_2$ $\hat{p}_2$	$\hat{u}_1$ $\hat{p}_1$
$\hat{\hat{u}}_1$ $\hat{\hat{p}}_1$	
$\vdots$	

Where to stop?

Asymmetric prescriptive/descriptive approach

- Bayesian approach (Raiffa, Kadane, Larkey…)
  - Prescriptive advice to one party conditional on a (probabilistic) description of how others will behave
  - Treat the other participant's decisions as uncertain

$$d^* = \text{argmax}_{d \in \mathcal{D}} \sum_{a \in \mathcal{A}} \int u_D(c) \pi_D(c|a, d) p_D(a|d) dc$$



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- **Simultaneous Games**
- Applications to Auctions
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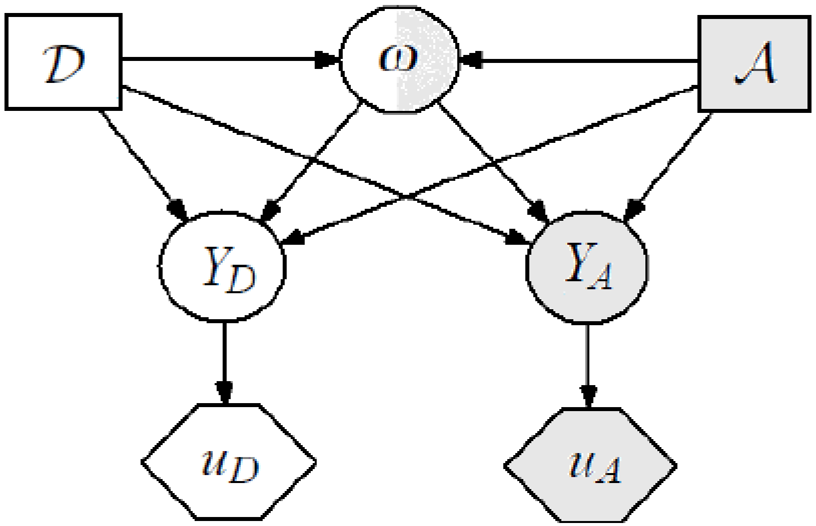
Simultaneous games

- Decisions are made without knowing each other's decisions



$$\psi_D(d,a) = \int u_D(a,d,\omega) p_D(\omega|a,d) d\omega.$$

$$\psi_A(d,a) = \int u_A(a,d,\omega) p_A(\omega|a,d) d\omega$$

	a
d	$(\psi_D(d,a), \psi_A(d,a))$

Game Theory Analysis

- Common knowledge
  - Each knows expected utility of every pair (d, a) for both of them
  - Nash equilibrium: (d*, a*) satisfying

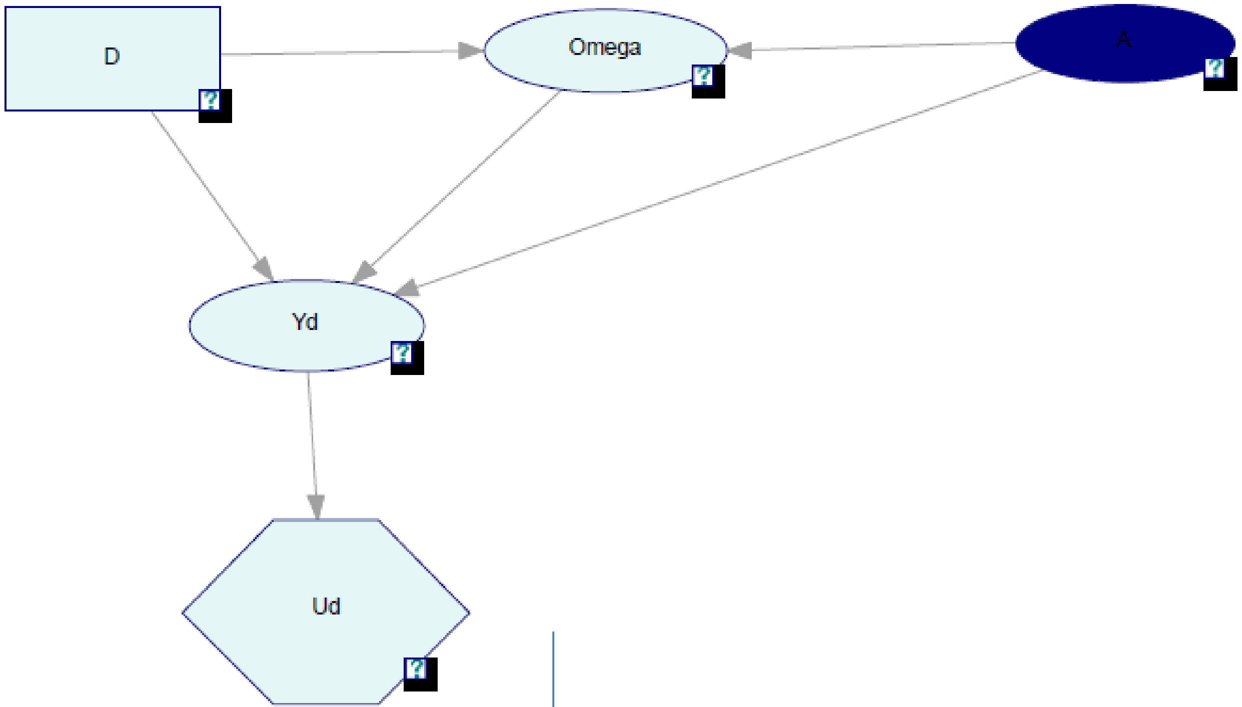
$$\psi_D(d^*, a^*) \geq \psi_D(d, a^*) \ \ \forall\, d \in \mathcal{D}$$

$$\psi_A(d^*, a^*) \geq \psi_A(d^*, a) \ \ \forall\, a \in \mathcal{A}$$

- When some information is not common knowledge
  - Private information. Types
  - Common prior over private information
  - Model the game as one of incomplete information
  - Bayes-Nash equilibrium

Supporting the Defender

- Defender's decision analysis



$$\max_d \psi_D(d) = \sum_a \psi_D(d,a) p_D(a) = \sum_a \left[\int u_D(a,d,\omega) p_D(\omega|a,d) d\omega \right] p_D(a)$$

Assessing $p_D(a)$

- Modeling the opponent
  - Nonstrategic adversary
  - Nasheq adversary
  - Level-k thinking adversary
  - Mirroreq adversary
  - Prospect adversary
- Model averaging

Assessing $p_D(a)$

- Non-strategic adversary

$$(p_1, \ldots, p_n)|a_i, d_j, \omega \sim \mathcal{D}(\alpha_1^{ij\omega}, ..., \alpha_n^{ij\omega})$$

$$(p_1, \ldots, p_n)|a_i, d_j, \omega, \text{data} \sim \mathcal{D}(\alpha_1^{ij\omega} + n_1^{ij\omega}, ..., \alpha_n^{ij\omega} + n_n^{ij\omega})$$

$$p_D^{NS}(a_k) = \frac{\alpha_k^{ij\omega} + j_k^{ij\omega}}{\sum_l (\alpha_l^{ij\omega} + j_l^{ij\omega})}$$

$$\max_d \sum_k \psi(d, a_k) p_D^{NS}(a_k)$$

Assessing $p_D(a)$

- Nasheq adversary

$$(U_A, P_A) \qquad (U_D, P_D)$$

$$(\psi_D^\theta(d,a), \psi_A^\theta(d,a)) \qquad \psi_D^\theta(d,a) = \int u_D^\theta(a,d,\omega) p_D^\theta(\omega|a,d) d\omega. \qquad (d^N(\theta), a^N(\theta))$$

$$p_D^N(a) = E_{\mathcal{P}}(a^N(\theta))$$

$$\max_d \sum_a \psi_D(d,a) p_D^N(a).$$

Assessing $p_D(a)$

- Level-k thinking adversary

$$d^* = \arg\max_d \left[\sum_a \psi_D(d,a) p_D(a) \right]$$

$$a^* = \arg\max_a \left[\sum_d \psi_D(d,a) p_A(d) \right] =$$

$$(U_A, P_A(\cdot|\cdot), P_A)$$

$$\arg\max_a \left[\sum_d \int u_A(d,a,\omega) p_A(\omega|a,d) d\omega \right] p_A(d)$$

$$A|D \sim \arg\max_a \sum_d \left[\int U_A(d,a,\omega) P_A(\omega|a,d) d\omega \right] P_A(d).$$

$$D|A^1 \sim \arg\max_d \sum_a \left[\int U_D(d,a,\omega) P_D(\omega|a,d) d\omega \right] P_D(a)$$

Assessing $p_D(a)$

- Level-k thinking adversary

Repeat from $i = 1$

Find $P_{D^{i-1}}(A^i)$ by solving

$$A^i \mid D^i \sim \arg\max_{a \in \mathcal{A}} \sum_{d \in \mathcal{D}} \left[\int U_A^i(a, d, \omega) P_A^i(\omega | a, d) d\omega \right] P_A^i(D^i = d)$$

$$\text{with } (U_A^i, P_A^i(.|.), P_A^i) \sim F^i$$

Find $P_A^i(D^i)$ by solving

$$D^i \mid A^{i+1} \sim \arg\max_{d \in \mathcal{D}} \sum_{a \in \mathcal{A}} \left[\int U_D^i(a, d, \omega) P_D^i(\omega | a, d) d\omega \right] P_D^i(A^{i+1} = a)$$

$$\text{with } (U_D^i, P_D^i(.|.), P_D^i) \sim G^i$$

$i = i + 1$

Assessing $p_D(a)$

- Level-k thinking adversary
  - A level-0 Daphne acts completely randomly (non strategically).
  - A level-1 Daphne chooses her alternative optimally, but assumes that Apollo acts randomly, thus being level-0, as was done in Section 3.1.
  - A level-2 Daphne assumes that Apollo is a level-1 Apollo thinker, who assumes she is a level-0 thinker. Daphne stops at $i = 1$ in the hierarchy, with F^1.
  - A level-3 Daphne assumes that she faces a level-2 adversary: Apollo's calculation assumes she is a level-1 thinker, who thinks about his objectives. Daphne stops at $i = 1$ in the hierarchy, with G^1.
  - A level-4 Daphne assumes she is facing a level-3 adversary: Apollo takes strategic account of what he thinks she thinks he thinks that she thinks. Daphne stops at $i = 2$ in the hierarchy, with F^2.
  - And so forth.

Assessing $p_D(a)$

- Mirroreq adversary

$$(U_A, P_A(\cdot|\cdot), P_A(\cdot)) \qquad (U_D, P_D(\cdot|\cdot), P_D(\cdot))$$

$$d^M(\theta) = \arg\max_d \sum_d \left[\int U_D^\theta(d,a,\omega) P_D^\theta(\omega|a,d) d\omega \right] p_D^M(a)$$

$$p_D^M = E_{\mathcal{P}} \left(\arg\max_a \sum_d \left[\int U_A^\theta(d,a,\omega) P_A^\theta(\omega|a,d) d\omega \right] P_A^{\theta M}(d) \right).$$

$$\max_d \sum_a \psi_D(d,a) p_D^M(a)$$

Assessing $p_D(a)$

- Prospect adversary

$$\max_a \left[\sum_d \int v_A(d,a,\omega) w_A^1(p_A(\omega|a,d))d\omega\right] w_A^2(p_A(d))$$

$$(V_A, W_A^1, W_A^2, P_A(\cdot|\cdot), P_A)$$

$$A|D = \arg\max_a \left[\sum_d \int V_A(d,a,\omega) W_A^1(P_A(\omega|a,d))d\omega\right] W_A^2(P_A(d))$$

Assessing $p_D(a)$

- Reconciling adversaries. Model averaging

$$\max_d \sum_a \psi_D(d,a) \left(\sum q_i p_D^i(a) \right) = \max_d \sum q_i \left[\sum_a \psi_D(d,a) p_D^i(a) \right]$$

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Auctions

- A very old market mechanism:

  Women sales in old Babylonia

- Much in use today:
  - Public procurement
  - Sotheby's
  - Wholesale market (fish, flowers,…)
  - UMTS Licences in Europe
  - eBay, Yahoo,…
  - Electrical market in Spain…
  - B2B

Economic relevance

- Competition aids sellers to price objects when willingness to pay is not known.
- Competition as an incentive generator
- Design of new competitive mechanisms, e.g. e-markets, when other mechanisms do not work, e.g., because of the meed to match demand and supply
- Triple objective
  - Set the buyer
  - Set the price
  - Sell the object quickly

Basic concepts

- Aucitons have rules and bidders:
  Auctioneer sets up rules and knows potential bidders
  Open and closed
- Two new types of uncertainties:
  - Private value.
  - Common value.
- Uncertainty about what the others will doosed

- Predated by game theory…
- Rothkopf (2007)!!!!!!

Complete info auctions

All participants know the value of the object given by other participants

- n participants want to get the object
- Object is worth v_i for paticipant i. All of them ordered from biggest to smallest (best is v_1; worst is v_n)

Nash equilibria??

First price sealed bids

- Each player bids bi and places it in a sealed envelope
- Auctioneer opens up envelopes and reveals bids
- Winner is maximum bidder.
- If r winners, randomize among them
- Payoff function is Ui (b1,…, bn) =

  – 0, if bi < m
  – 1/r (vi-bi), if bi=m

  With m=max(b1,…,bn)

First price sealed bids

- Player i bids bi <= vi. Consider two cases

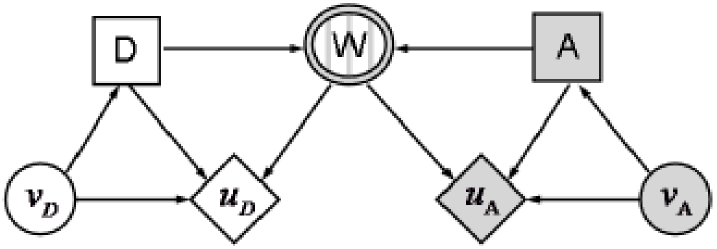
1. If v1>v2, player 1 wins bidding b1 such that v2<b1<v1
2. There are ties, all finalists bid bi=vi

We have:

1. There is no Nasheq, but proposed solution is, approximately a Nasheq.
2. (v1, v2,…, vn) is Nasheq

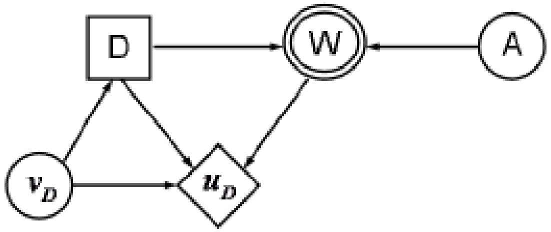
First price sealed-bid auctions

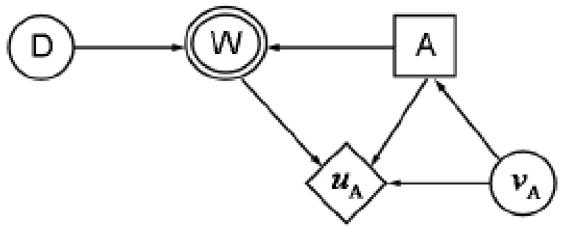
Problem in parallel

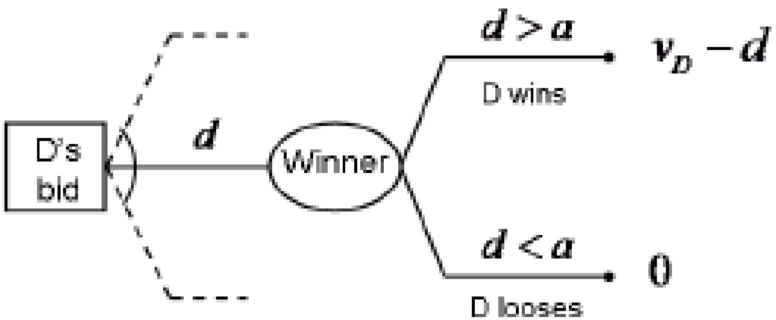


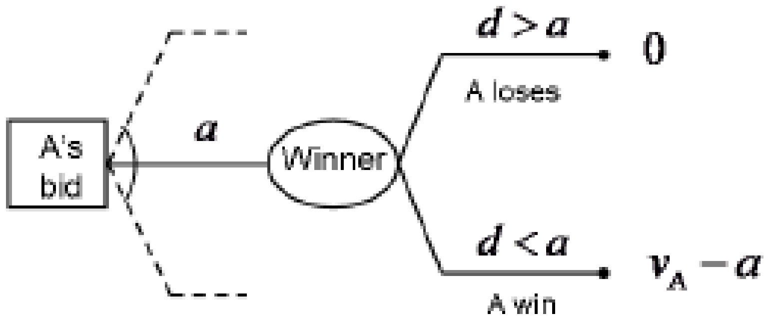
A's problem

D's problem







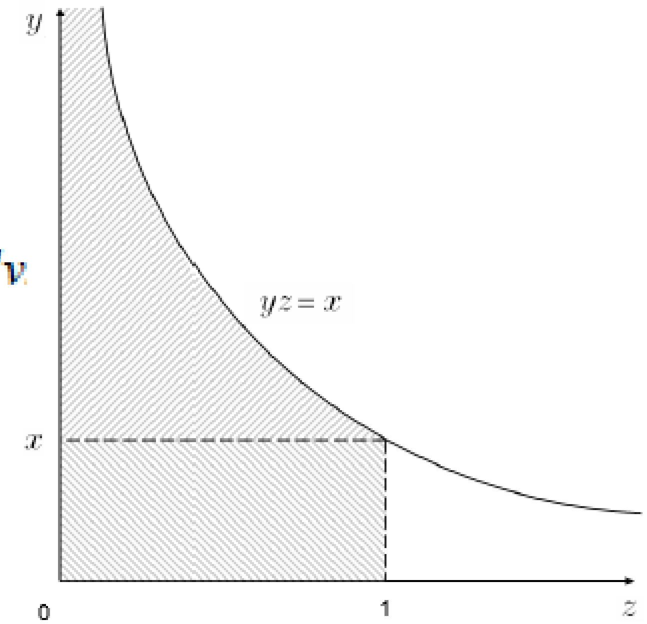


Non Strategic

$$x^* = \text{argmax}_{x \in \mathbb{R}^+} (x_0 - x) F(x)$$

$$F(y) = \mathbb{P}[PV \leq y] = \int_0^\infty \int_0^{y/v} g_1(v) g_2(p)\, dp\, dv$$

$$\mathbb{E}_H[(X_0 - x)F(x)] = (\mu - x)F(x)$$



Nasheq

Nasheq suggests bidding your true value

ARA suggests underbidding, in consonance with experimental findings

$$x^* = \text{argmax}_{x \in \mathbb{R}^+} (x_0 - x) F(x).$$

Second price auctions (Vickrey). Nasheq and ARA coincid: bid true value

Level-k

- Level 0
- Level 1
- Level 2

$y^* = \text{argmax}_{y \in \mathbb{R}^+} (y_0 - y) G_C(y)$

1. Repeat from $j = 1$ to J

   Sample $y_0^j \sim H$

   Solve $y_j^* = \text{argmax}_{y \in \mathbb{R}^+} (y_0^j - y) G_B(y)$
2. Set $\hat{F}_J(x) = \frac{1}{J} \sum_{j=1}^{J} I(y_j^* \leq x)$
3. Solve $x^* = \text{argmax}_{x \in \mathbb{R}^+} (x_0 - x) \hat{F}_J(x)$

$Y^* = \text{argmax}_{y \in \mathbb{R}^+} (Y_0 - y) G_B(y)$

0. Repeat from $i = 1$ to I

   Sample $V_i \sim G_1$

   Sample $P_i \sim G_2$

   Set $Z_i = P_i V_i$
1. Set $\hat{G}_I(y) = \frac{1}{I} \sum_{i=1}^{I} I(Z_i^* \leq y)$
2. Repeat from $j = 1$ to J

   Sample $y_0^j \sim H$

   Solve $y_j^* = \text{argmax}_{y \in \mathbb{R}^+} (y_0^j - y) \hat{G}_I(y)$
2. Set $\hat{F}_J(x) = \frac{1}{J} \sum_{i=1}^{J} I(y_j^* \leq x)$
3. Solve $x^* = \text{argmax}_{x \in \mathbb{R}^+} (x_0 - x) \hat{F}_J(x)$.

$x^* = \text{argmax}_{x \in \mathbb{R}^+} (x_0 - x) F(x)$

Level-k

- Dirichlet processes
- Choosing the level

BayesNasheq

- Symmetric case $\mathbb{E}[P] = [v - b(v)]\mathbb{P}[b(v) \text{ wins}].$ $\mathbb{E}[P] = [v - b(v)]F(v)^{n-1}$

$$b(v) = \frac{\int_0^v z(n-1)F'(z)F(z)^{n-2}\,dz}{F(v)^{n-1}}$$

- Asymmetric case

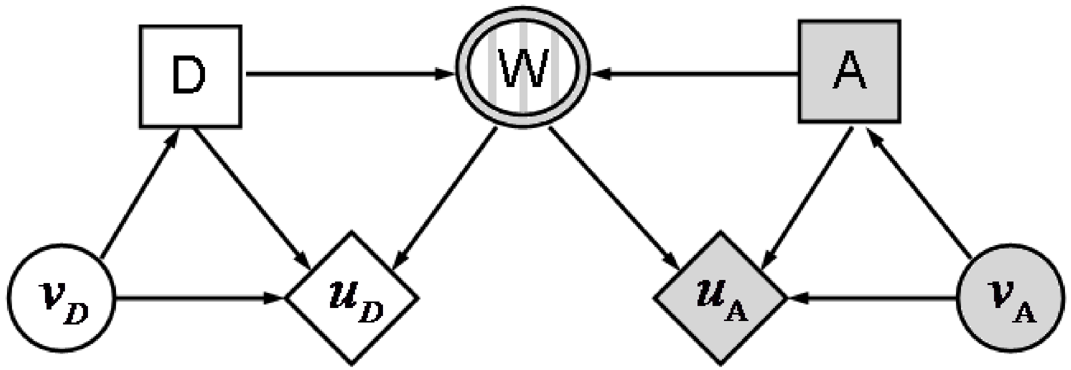
$$B^* = \text{argmax}_{b \in \mathbb{R}^+} (b_0 - b)F(b) \qquad C^* = \text{argmax}_{c \in \mathbb{R}^+} (c_0 - c)G(c)$$

$$B^* = \text{argmax}_{b \in \mathbb{R}^+} (B - b)F(b) \sim G$$
$$C^* = \text{argmax}_{c \in \mathbb{R}^+} (C - c)G(c) \sim F.$$

Bidding in a two-person sealed-bid Auction

- Two sealed bids, the highest one wins
  - Simultaneous decision problem
- The standard Game Theory Analysis
  - D knows v_D but A does not: $\mathrm{p_A}(v_D)$
  - A knows v_A but D does not: $\mathrm{p_D}(v_A)$
  - Common knowledge assumption

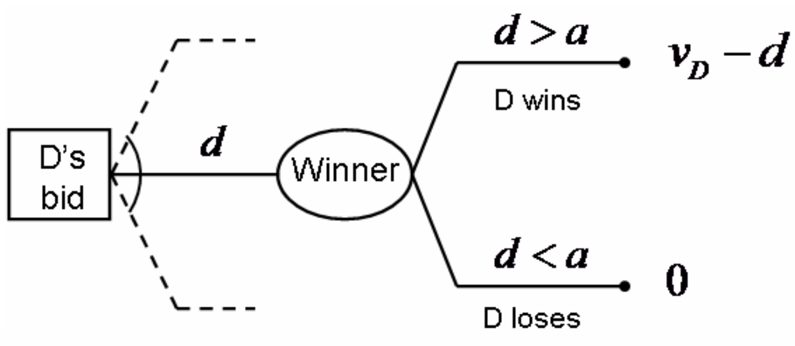
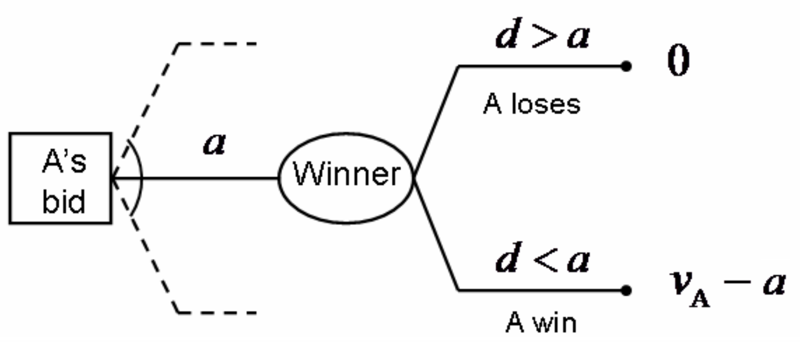
$$\mathrm{p_A}(v_D) = \mathrm{p}(v_D)$$
$$\mathrm{p_D}(v_A) = \mathrm{p}(v_A)$$



  - Bayesian Nash Eq. (Harsanyi)
  - Is it rational that players' beliefs about the opponent's object value will be disclosed??

Rothkopf (2007)

Supporting D

D's problem	D's analysis of A's problem
	

$$\max_d u_D(v_D - d)\, \mathbb{P}_D(d > \underline{\underline{a}}|d)$$

$$\max_a \widehat{u}_A(\widehat{v}_A - a)\, \underbrace{\widehat{\mathbb{P}}_A(a > \underline{\underline{d}}|a)}_{\int_{-\infty}^{a} \widehat{\pi}_A(d)\, \mathrm{d}d}$$

A's prob. of winning given his bid a

?? $(\widehat{u}_A, \widehat{v}_A, \widehat{\mathbb{P}}_A)$

$d \sim \widehat{\pi}_A$

The assessment problem (III)

- Assessment of $d \sim \widehat{\pi}_A$
- D’s analysis of A’s analysis of D’s problem
  - It leads to a infinite analysis of previous analysis…
- Avoiding infinite regress
  - Available past statistical data (Capen et al, Keefer et al)
  - Expert knowledge
  - Non-informative distribution
  - Heuristic based elicitation (*)
- Heuristic elicitation $\widehat{\pi}_A(d)$
  - Identification of relevant variables in which A can base his assessment of D’s bid $d \sim \widehat{\pi}_A$

Relevant variables

- Auctioned object (true) value for
  - D: v_D
  - A: **?**
- D's analysis of A's problem (D's guessed values)
  - A's value: $v_A \sim V_A$
  - A's guess of …
    - D's value: $\hat{v}_D$
    - D's guess of A's value: $\hat{v}_A$

Used by A to guess D's bid

$$d \sim \widehat{\pi}_A$$

as a function of $\hat{v}_D$ and $\hat{v}_A$

- Variables that D needs to assess

v_D	v_A
$\hat{v}_D$	$\hat{v}_A$

The assessment solution: An heuristic elicitation approach

- D's analysis of A's problem
- Helping D in the assessment of $\widehat{\pi}_A(d)$
  - D's analysis of A's analysis about D's bid

$$\widehat{\pi}_A(d) = N\left(\min\left(\alpha\, \hat{v}_D, \beta\, \hat{v}_A\right), \sigma\right) \qquad \alpha, \beta \in (0,1)$$ truncated

$$(\alpha, \hat{v}_D, \beta, \hat{v}_A, \sigma) \sim \Pi_A$$

- Assess $F = (V_A, \Pi_A)$
  - D's uncertainties in her analysis of A's problem

Heuristic elicitation approach

Assuming D and A are risk neutral

<u>Supporting D</u>

$$d^* = \arg\max_d (v_D - d)\,\mathrm{P}_D(d > a \mid d)$$

v_D $\quad$ $\mathrm{P}_D(d > a \mid d)$

r. v.

$\pi_D(a)$?? $\qquad a^* = \arg\max_a (v_A - a)\,\mathrm{P}_A(a > \hat{d} \mid a)$

$v_A \sim V_A$ $\qquad \hat{\pi}_A(\hat{d})$

$\hat{d} \sim \mathrm{N}(\min(\alpha\hat{v}_D, \beta\hat{v}_A), \sigma)$

$(\hat{v}_D \quad \hat{v}_A \quad \alpha \quad \beta \quad \sigma) \sim \Pi_A$

A's expectations about her bid

Truncated Normal distribution: $\hat{d} \in (0, \hat{v}_D)$

$\hat{v}_D \sim \hat{V}_D$ $\qquad \hat{\pi}_D(\hat{a})$

$\hat{v}_A \sim \hat{V}_A$

Computation of D's bid of max expected utility

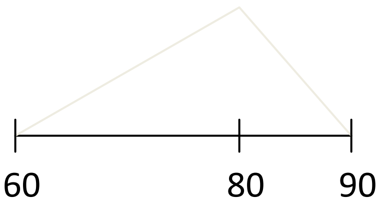
Step 1 $$\hat{d} \sim \mathrm{N}(\min(\alpha\hat{v}_D, \beta\hat{v}_A), \sigma)$$

Step 2 $$A^* \sim \arg\max_a (V_A - a)\mathrm{P}_A(a > \hat{d} \mid a)$$

Step 3 $$d^* = \arg\max_d (v_D - d)\mathrm{P}_D(d > A^* \mid d)$$

Numerical example: D's assessments

- Value of the object for D (known)
  - $v_D = 100$ Euros
- D's prediction of A's bid $\pi_D(a)$
  - $v_A \sim V_A$ A's valuation of the object
    - D assesses mode (80), max (90) and min (60)
    - We fit a triangular distribution



$\hat{\pi}_A(\hat{d})$

Step 1

  - $\hat{v}_D \sim \hat{V}_D$ A's estimation of D's valuation v_D
    - D assesses that it is 100 Euros higher than v_A with a 5 Euros error
    - $\hat{V}_D \mid v_A = v_A + 100 + U(-5,5)$
  - $\hat{v}_A \sim \hat{V}_A$ D's estimation of v_A as thought by A
    - D assesses that it is 50 Euros lower than v_A with a 5 Euros error
    - $\hat{V}_A \mid v_A = v_A - 50 + U(-5,5)$
  - Profit proportion bidding expected behavior
    - $1 - \alpha = 0.3 + U(-0.05, 0.05)$
    - $1 - \beta = 0.3 + U(-0.05, 0.05)$
  - $\sigma = 1$ D's confidence on heuristic and assessments

Step 1
D's heuristic elicitation of A's beliefs on how she will bid

Step 1 $$\hat{D} \mid v_A, v_D, \alpha, \beta \sim \mathrm{N}(\min(\alpha \hat{v}_D, \beta \hat{v}_A), \sigma)$$

Step 2 $$A^* \sim \arg\max_a (V_A - a) \mathrm{P}_A(a > \hat{d} \mid a)$$

Step 3 $$d^* = \arg\max_d (v_D - d) \mathrm{P}_D(d > A^* \mid d)$$

Step 2
Sampling from A*
D's predictive distribution on A's bid

Step 1	$\hat{D} \mid v_A, v_D, \alpha, \beta \sim \mathrm{N}(\min(\alpha \hat{v}_D, \beta \hat{v}_A), \sigma)$
Step 2	$A^* \sim \arg\max_a (V_A - a) \mathrm{P}_A(a > \hat{d} \mid a)$
Step 3	$d^* = \arg\max_d (v_D - d) \mathrm{P}_D(d > A^* \mid d)$

MC simulation

- Repeat for $i = 1, \ldots, n$

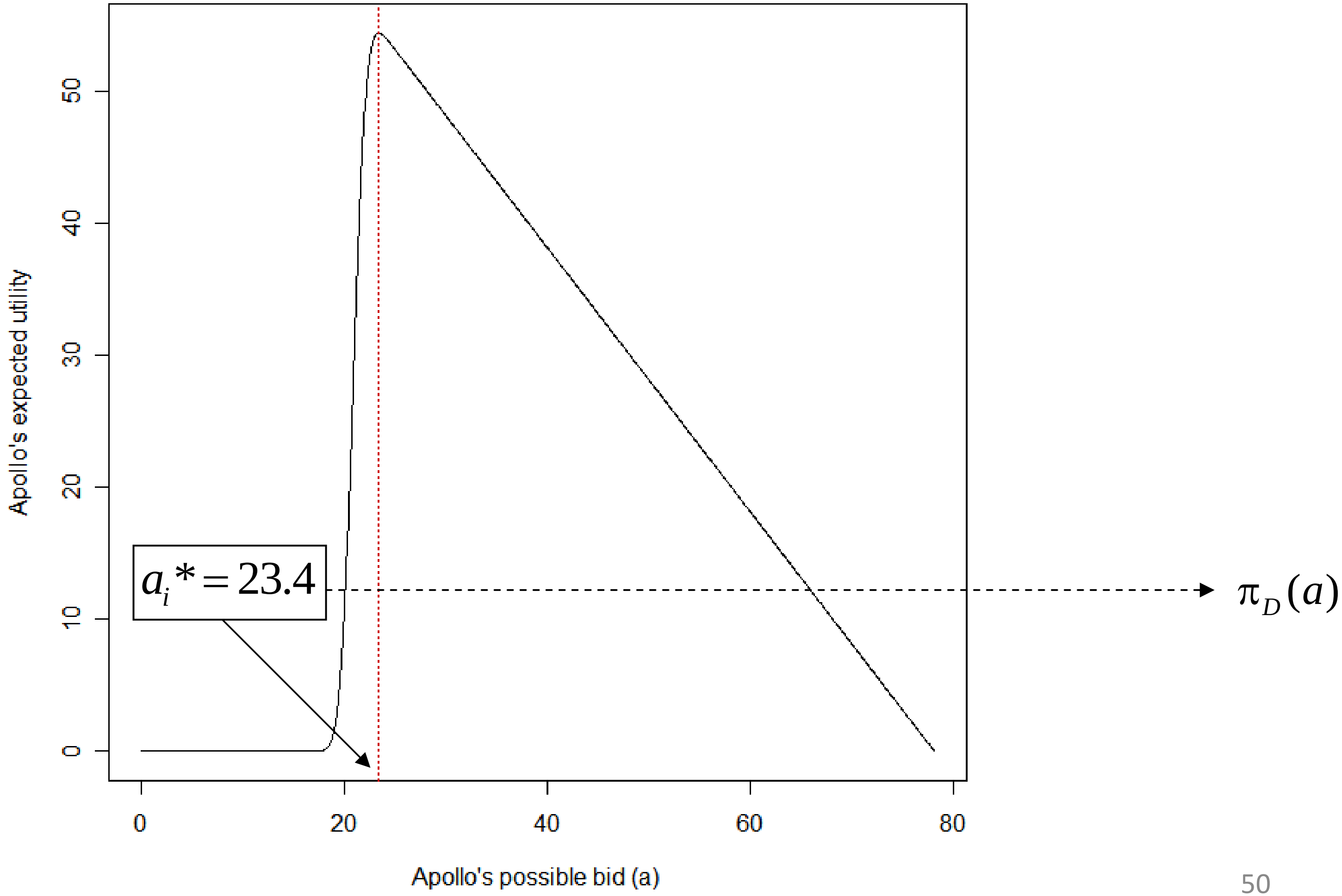
1. $v_A^i \sim V_A$
2. $(\alpha^i, \hat{v}_D^i, \beta^i, \hat{v}_A^i, \sigma^i) \sim \Pi_A \mid v_A^i$

   $\hat{d}_i \sim N\left(\min\left(\alpha^i\, \hat{v}_D^i, \beta^i\, \hat{v}_A^i\right), \sigma^i\right)$
3. Solve $a_i^* = \text{argmax}_a (v_A^i - a)\, \mathbb{P}_{\hat{\pi}_A^i}(\hat{d}_i < a | a)$ → (exact) CPF of the Truncated Normal in a

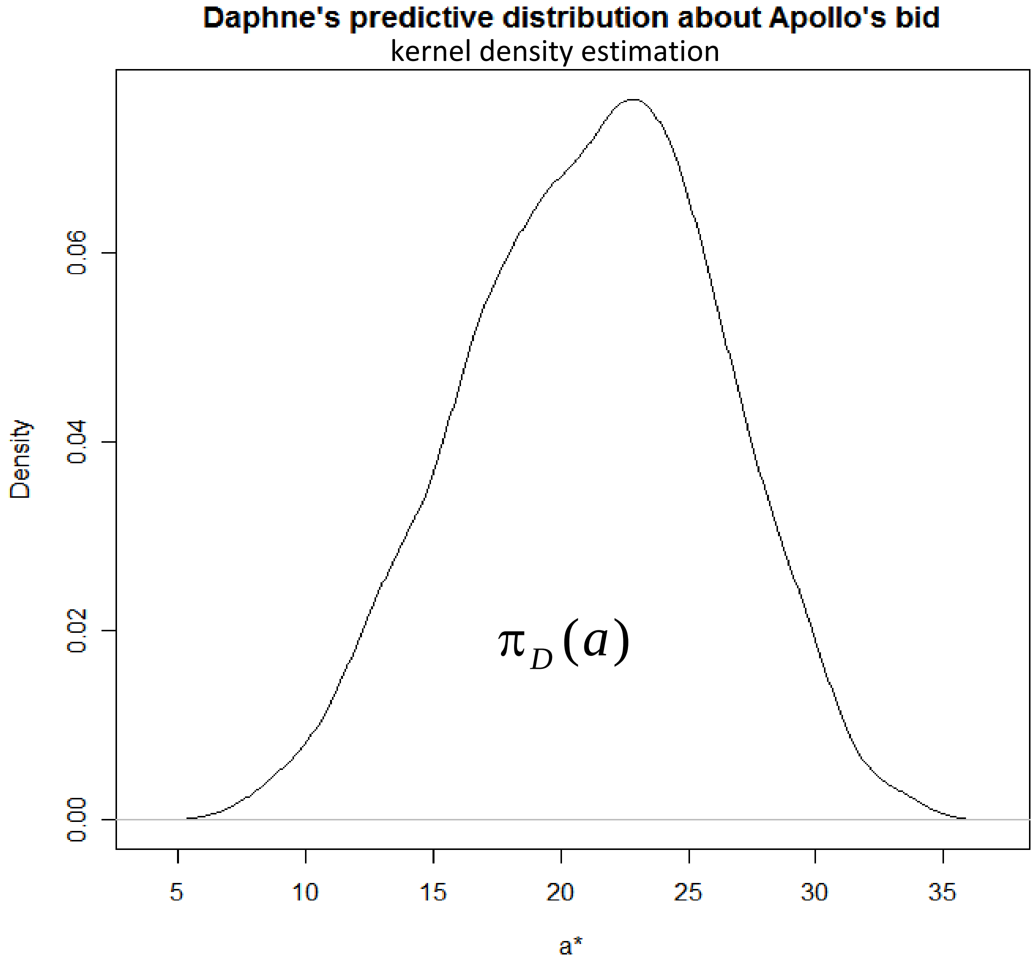
- Collect $\{a_i^*,\ i = 1, \ldots, n\}$
- Note the pdf of A* is $\pi_D(a)$

Step 2: Solving Apollo's optimization problem at the i-th iteration

$$a_i^* = \arg\max_a (v_A^i - a) P_A(a > \hat{d}_i \mid a)$$



Step 2
Estimation of $\pi_D(a)$ from $\{a_i^*, \ i = 1, \ldots, n\}$



D's prediction of Apollo's bid

Step 3
Solving D's optimization problem

Step 1 $\quad \hat{D} \,|\, v_A, v_D, \alpha, \beta \sim \mathrm{N}(\min(\alpha \hat{v}_D, \beta \hat{v}_A), \sigma)$

Step 2 $\quad A^* \sim \arg\max_a (V_A - a)\mathrm{P}_A(a > \hat{d} \,|\, a)$

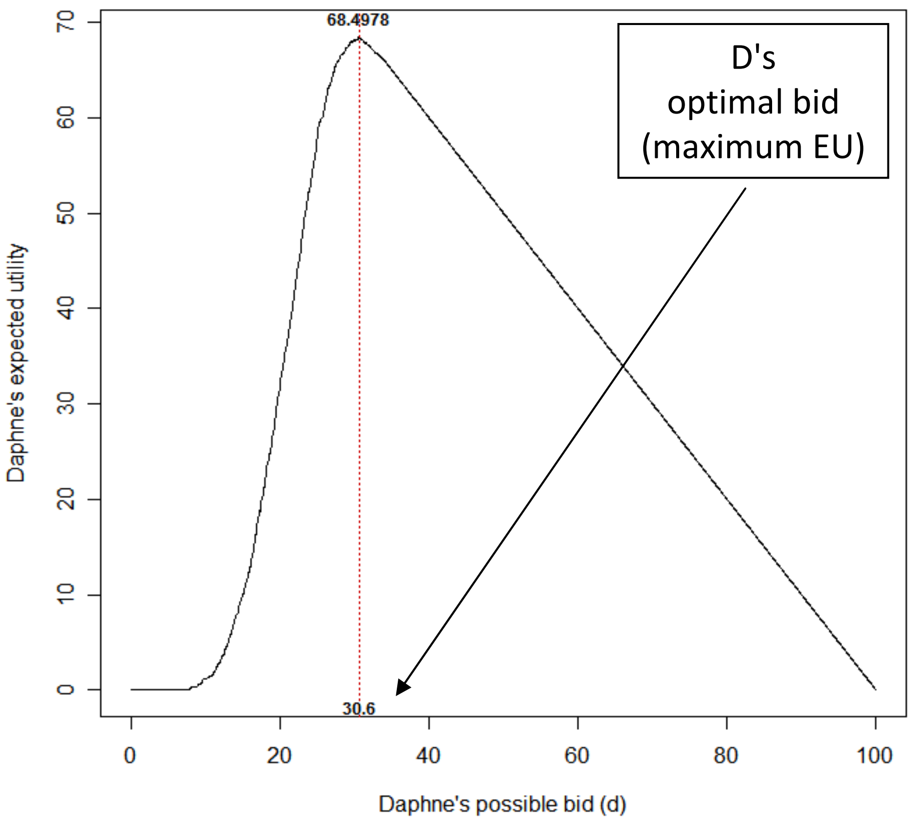
Step 3 $\quad d^* = \arg\max_d (v_D - d)\mathrm{P}_D(d > A^* \,|\, d)$

Step 3
Solving D's maximum expected utility bid

- D’s optimal bid

$$d^* = \arg\max_d \, (v_D - d) \, \hat{\mathbb{P}}_D(d > A^* | d)$$

where $\hat{\mathbb{P}}_D(d > A^* | d) \approx \#\{d > a_i^*\}/n$ MC estimation

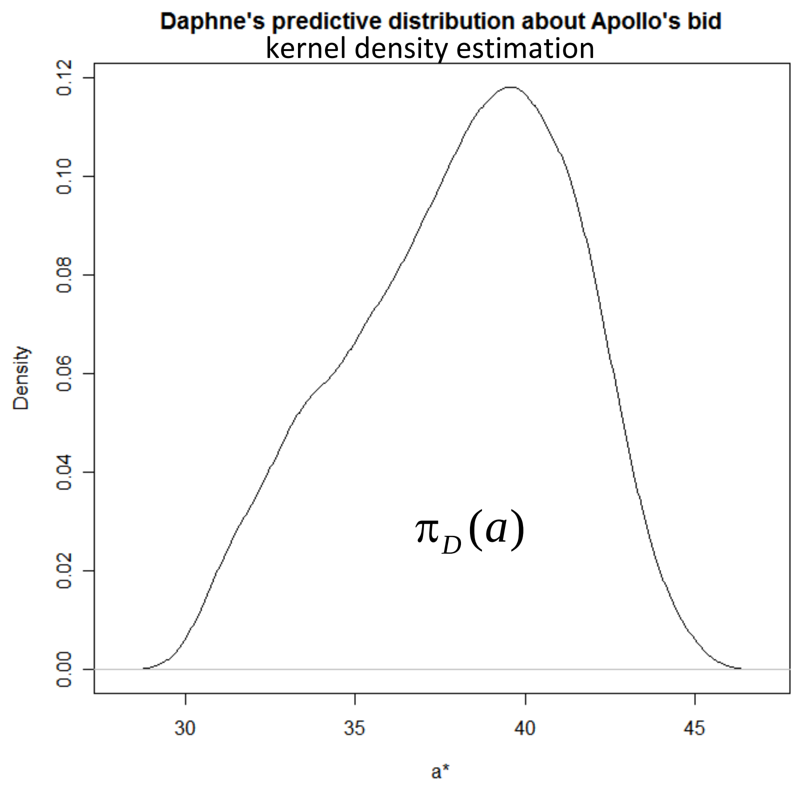


MC estimation of E.U.(d)
&
MC optimization

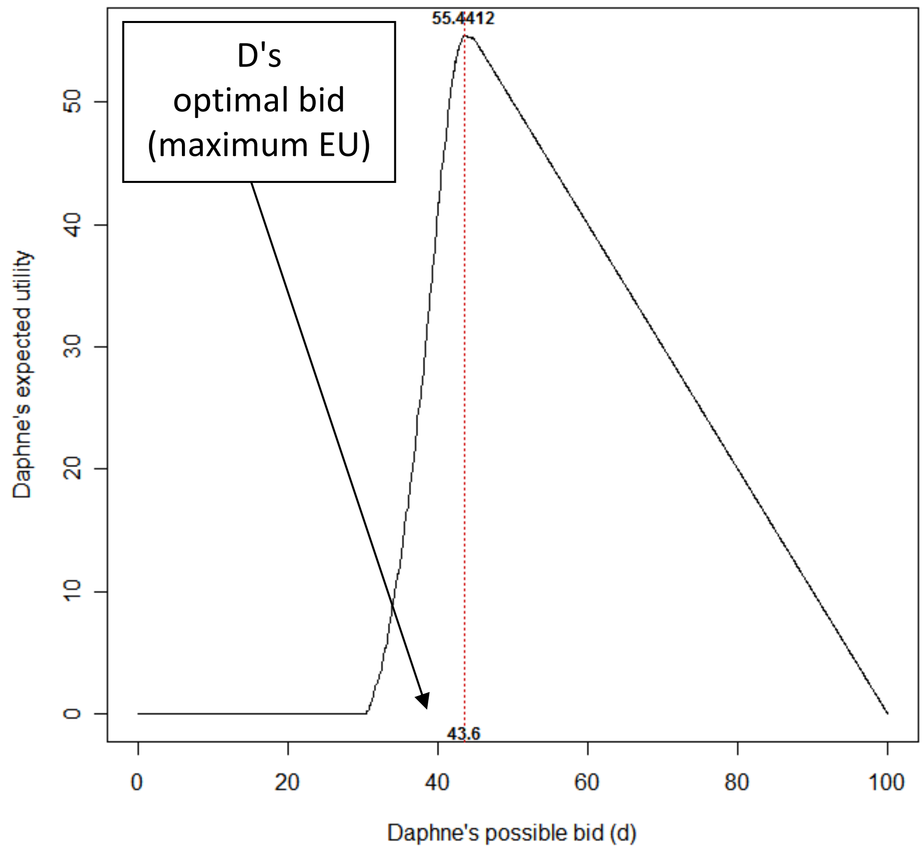
D's optimal bid: d* = **30.6**
with (estimated) Expected Utility = **68.5**

What happens when σ gets larger?

- Same numerical example as before but with $\sigma = 100$
  - D's optimal bid **d*= 43.6** with EU **55.4**



D’s prediction of Apollo's bid



Analysis of Step 2

Solving Apollo's optimization problem at the i-th iteration

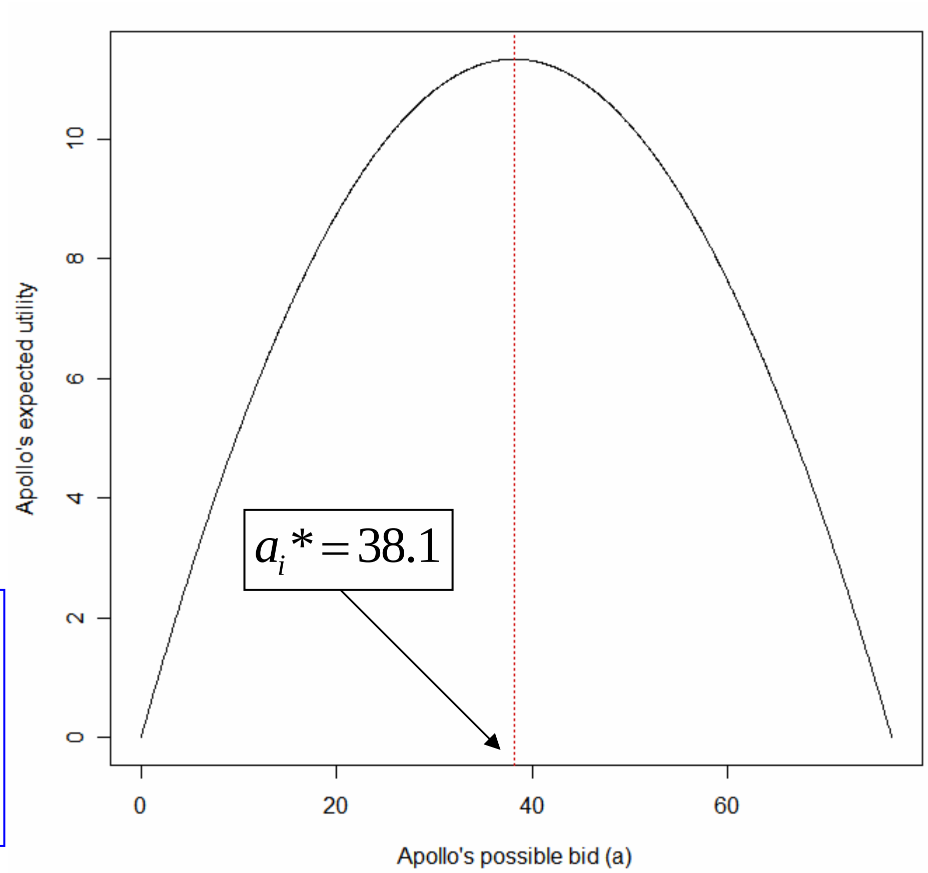
$$\hat{d}_i \sim \mathrm{N}(\min(\alpha\hat{v}_D, \beta\hat{v}_A), \sigma = 100)$$

$$\pi_A(\hat{d}_i) \approx cte$$

$$\hat{d}_i \approx U(0, \hat{v}_D^i)$$

$$a_i^* = \arg\max_a (v_A^i - a)\mathrm{P}_A(a > \hat{d}_i \mid a)$$

$$\arg\max_a (v_A^i - a)\frac{a}{\hat{v}_D^i}$$



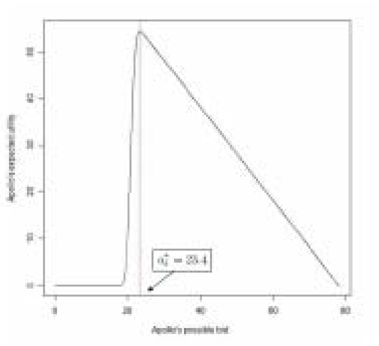
If $\frac{v_A^i}{2} < \hat{v}_D^i$, $\quad a_i^* = \frac{v_A^i}{2}$ with $v_A^i \sim V_A$ & $\hat{v}_D^i \sim \hat{V}_D$

else $\quad a_i^* = \hat{v}_D^i$

This is not the complete non-informative case for d

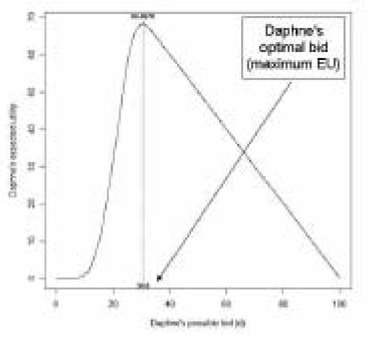
The non-informative distribution for d is $\hat{d} \propto 1$ which do not use info from $\begin{cases} \hat{v}_D \sim \hat{V}_D \\ \hat{v}_A \sim \hat{V}_A \end{cases}$

with solution $a_i^* = v_A^i / 2 \quad v_A^i \sim V_A$



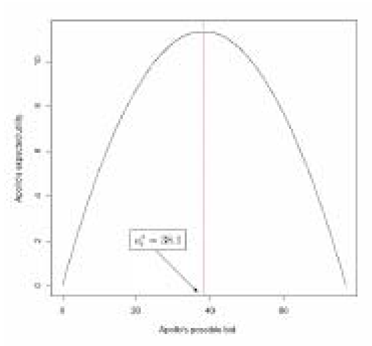
(a) Solving Apollo's optimization problem at the i-th iteration

(b) Kernel density estimation of $\pi_D(a)$

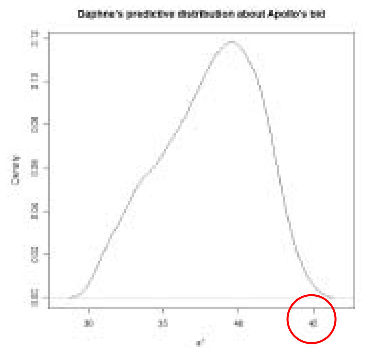


(c) Daphne's expected utility function

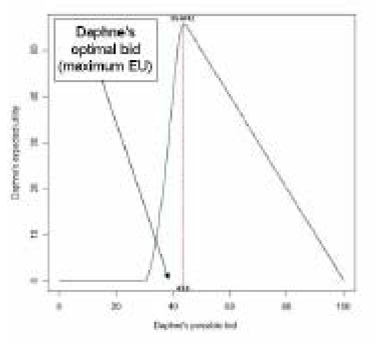
Figure 13: Numerical Example



(a) Solving Apollo's optimization problem at the i-th iteration



(b) Kernel density estimation of $\pi_D(a)$



(c) Daphne's expected utility function

Figure 14: Noninformative case ($\sigma = 100$)

Noninformative case

when D does not provide us

with enough information

to asses a prior over $\hat{d}$.

or

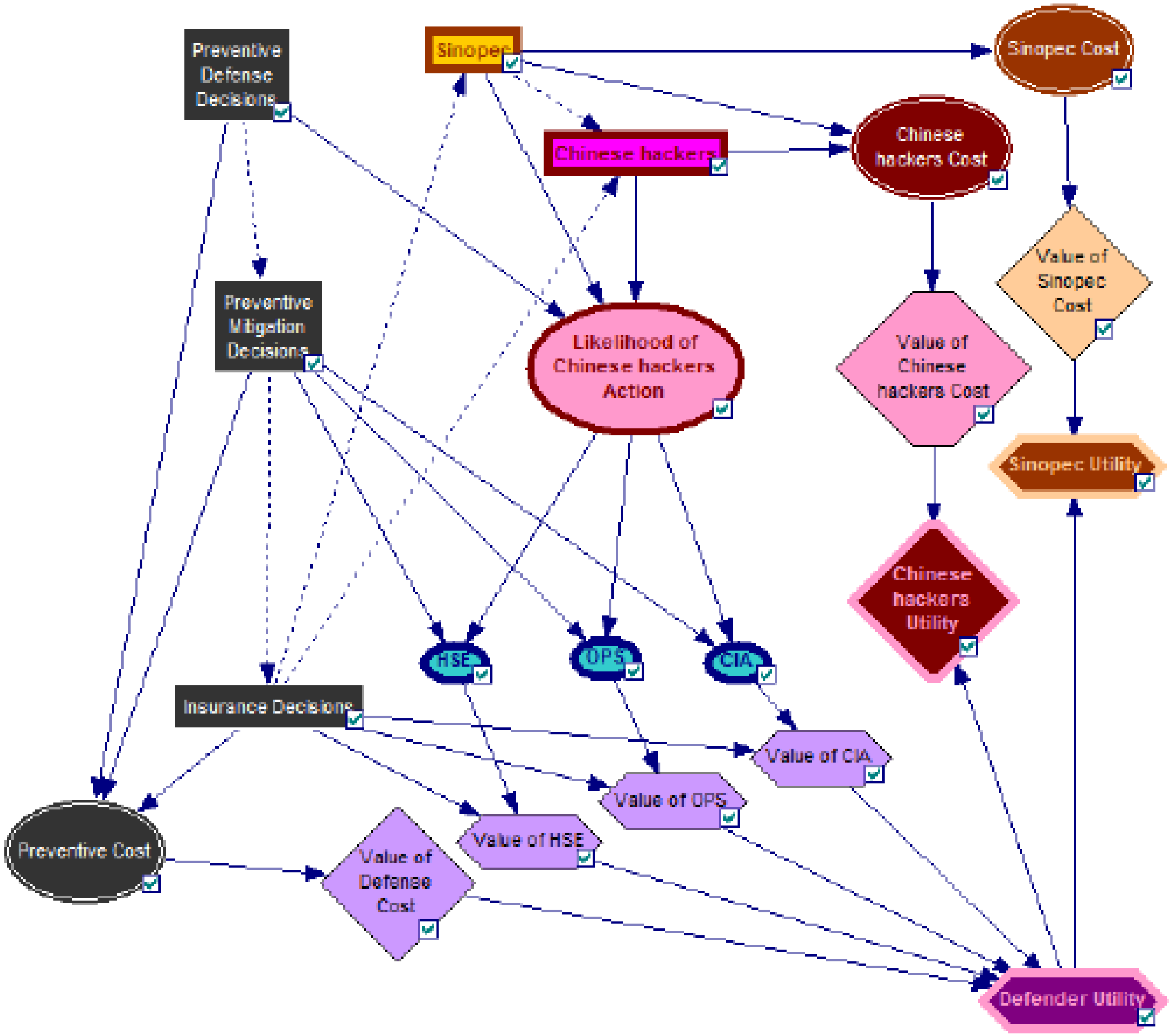
D is not confident

the heuristic model

Agenda

- Adversarial Risk Analysis: Intro
- Simultaneous Games
- Applications to Auctions
- **Conclusions**

- New perspective on conflict modeling
- Illustrated with two person risk neutral first price sealed auctions
- Multiperson
- Risk averse
- Other auctions
- Insurance



Preventive Defense Decisions
Sinopec
Sinopec Cost
Chinese hackers
Chinese hackers Cost
Value of Sinopec Cost
Preventive Mitigation Decisions
Likelihood of Chinese hackers Action
Value of Chinese hackers Cost
Sinopec Utility
Chinese hackers Utility
HSE
OPS
CIA
Insurance Decisions
Value of CIA
Value of OPS
Value of HSE
Preventive Cost
Value of Defense Cost
Defender Utility

- Thanks!!!
- Grazie, Gracias, Obrigado

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