

# Adversarial Risk Analysis Models for Urban Security Resource Allocation

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# ARA for Urban Security Resource Allocation

- Security
- Urban security from a modelling perspective
- Adversarial Risk Analysis
- ARA for Urban Security Resource Allocation
- Discussion

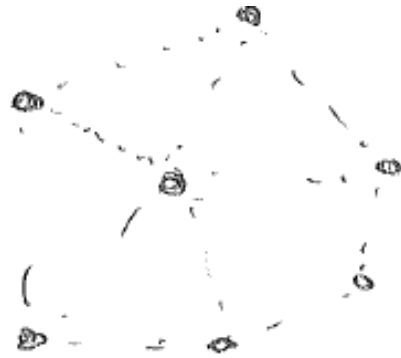
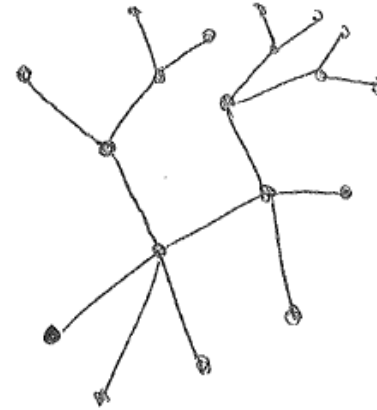
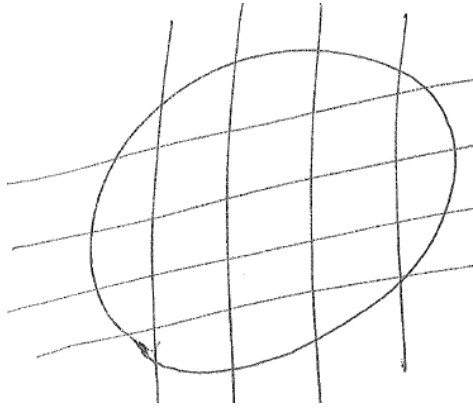
# Security

- One of 'The World's (23) Biggest Problems' (Lomborg, 2008)
  - Arms proliferation
  - Conflicts
  - Corruption
  - Terrorism
  - Drugs
  - Money laundering

# Security

- One of FP7 priorities
- SECONOMICS (2012-2015)
  - Anadolu Airport
  - Barcelona underground
  - National Grid, UK

# Security



# Urban Security from a modelling perspective

- Criminology
- Becker (1968) Economic theory of delict
- Clarke and Cornish (1986) Situational crime prevention. The reasoning criminal
  - Rational Choice in criminology
  - Routine activities theory
  - Delictive pattern theory
  - Problem-oriented policing
- Displacement theory
- Policing performance measures

# Urban Security from a modelling perspective

- COMPSTAT (1994)
- Crime Mapping
- Patrol Car Allocation Models (Tongo, 2010)
- Police Patrol Area Covering Models (Curtin et al, 2007)
- Police Patrol Routes Models (Chawathe, 2007)
- ARMOR at LAX (CREATE, 2007, 2009, 2011)
  
- The Numbers behind NUMB3RS (Devlin, Lorden, 2007)

# Adversarial Risk Analysis

- S-11, M-11 lead to large security investments globally, some of them criticised
- Many modelling efforts to efficiently allocate such resources
- Parnell et al (2008) NAS review
  - Game theoretic approaches. Common knowledge assumption...
  - Decision analytic approaches. Forecasting the adversary action...
- Merrick, Parnell (2011) review approaches commenting favourably on Adversarial Risk Analysis



# Adversarial Risk Analysis

- A framework to manage risks from actions of intelligent adversaries (DRI, Rios, Banks, JASA 2009)
- One-sided prescriptive support
  - Use a SEU model
  - Treat the adversary's decision as uncertainties
- Method to predict adversary's actions
  - We assume the adversary is a *expected utility maximizer*
    - Model his decision problem
    - Assess his probabilities and utilities
    - Find his action of maximum expected utility
  - But other *descriptive* models are possible
- Uncertainty in the Attacker's decision stems from
  - *our* uncertainty about his probabilities and utilities
  - but this leads to a hierarchy of nested decision problems

(noninformative, heuristic, mirroring argument) vs (common knowledge)

# Adversarial Risk Analysis

- ARA applications to counterterrorism models (Rios, DRI, 2009, 2012 Risk Analysis)
  - Sequential Defend-Attack
  - Simultaneous Defend-Attack
  - Sequential Defend-Attack-Defend
  - Sequential Defend-Attack with private information
- Somali pirates case (Sevillano, Rios, DRI, 2012 Decision Analysis)
- Routing games (anti IED war) (Wang, Banks, 2011)
- Borel games (Banks, Petralia, Wang, 2011)
- Auctions (DRI, Rios, Banks, 2009; Rothkopf, 2007)
- Kadane, Larkey (1982), Raiffa (1982), Lippman, McCardle (2012)
- Stahl and Wilson (1994,1995)<sup>10</sup> D. Wolpert (2012)

# ARA for Urban Security. Basics

- City divided into cells  $(i,j)$
- Each cell has a value  $v_{ij}$
- Actors
  1. Defender, D, Police. Aims at maintaining value
  2. Attacker, A, Mob. Aims at gaining value

- D allocates resources to prevent
- A performs attacks
- D allocates resources to recover  
Plus other constraints

$$\sum_{ij} d_{ij}^1 \leq D_1$$
$$\sum_{ij} a_{ij} \leq A$$
$$\sum_{ij} d_{ij}^2 \leq D_2$$

# ARA for Urban Security. Basics

	1	2	3
1			
2			
3			

	1	2	3
1	1,0	0,8	0,6
2	0,4	0,2	0,5
3	0,7	0,9	1,0

$$\sum_{ij} d_{ij}^1 \leq 11 \text{ y } d_{ij}^1 \geq 0, d_{ij}^1 \text{ integer}$$

	1	2	3
1	1	0	1
2	1	3	1
3	1	1	2

	1	2	3
1	11	0	0
2	0	0	0
3	0	0	0

	1	2	3
1	1	0	1
2	0	1	0
3	0	0	0

	1	2	3
1	0	0	0
2	0	0	0
3	0	0	3

$$\sum_{ij} a_{ij} \leq 3 \text{ y } a_{ij} \geq 0, a_{ij} \text{ integer}$$

$$\sum_{ij} d_{ij}^2 \leq 11, d_{ij}^2 \geq 0, d_{ij}^2 \text{ integer}$$

$$\sum_{ij} d_{ij}^2 \leq \sum_{(k,h) \in (i,j)_m} d_{kh}^1$$

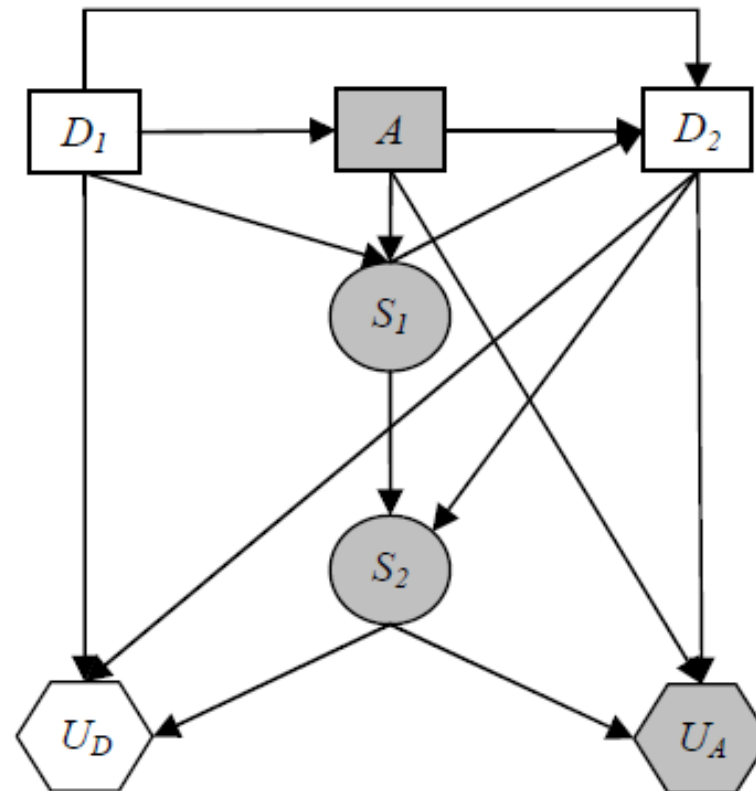
	1	2	3
1	$d_{11}^1$	$d_{12}^1$	$d_{13}^1$
2	$d_{21}^1$	$d_{22}^1$	$d_{23}^1$
3	$d_{31}^1$	$d_{32}^1$	$d_{33}^1$

	1	2	3
1	$d_{11}^2$	$d_{12}^2$	$d_{13}^2$
2	$d_{21}^2$	$d_{22}^2$	$d_{23}^2$
3	$d_{31}^2$	$d_{32}^2$	$d_{33}^2$

# ARA for Urban Security. Basics

At each cell, a  
coupled  
influence diagram

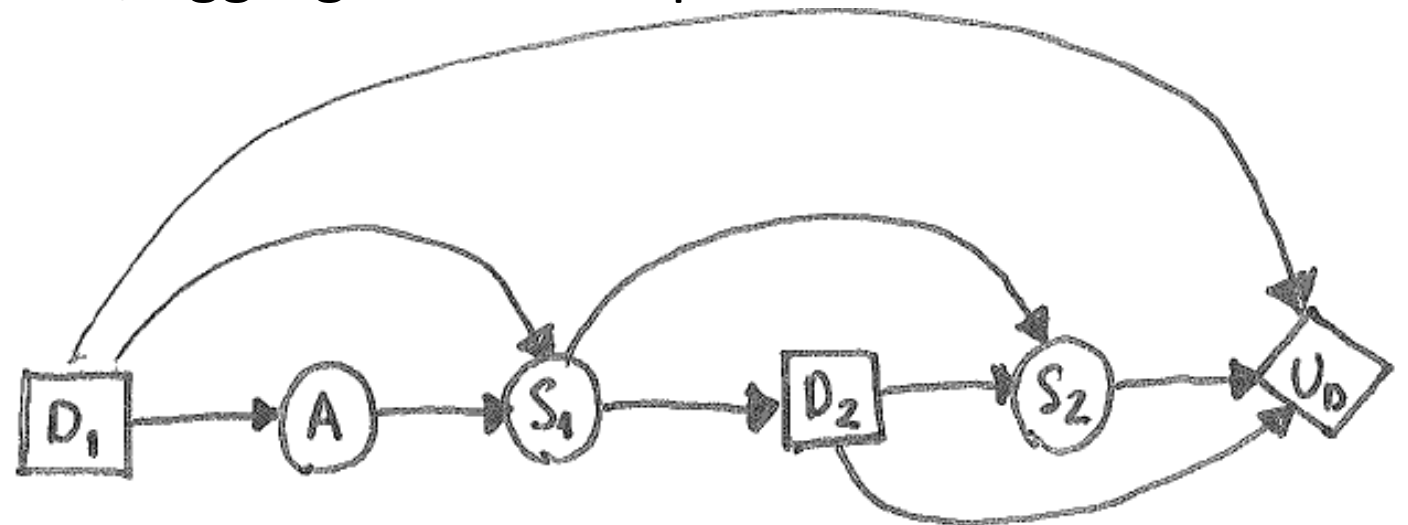
Cell decision making  
coordinated by  
constraints on  
resources



# ARA for Urban Security. Police dynamics

At each cell:

- Makes resource allocation  $d_{ij}^1$
- Faces a level of delinquency  $a_{ij}$ , with impact  $s_{ij}^1$
- Recovers as much as she can with resources  $d_{ij}^2$  with a level of success  $s_{ij}^2$
- Gets a consequence
- Aggregates utilities/Aggregates consequences



# ARA for Urban Security. Police dynamics

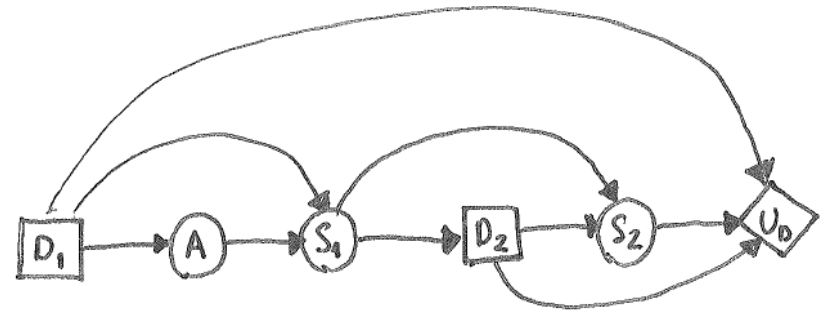
The assessments required from the defender are

- $p_D(a|d_1)$
- $p_D(s_1|a, d_1)$
- $p_D(s_2|s_1, d_2)$
- $u_D(d_1, s_2, d_2, v)$

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# ARA for Urban Security. Police dynamics

The Police solves sequentially



- At node  $U_D$ ,  $u_D(d_1, s_2, d_2, v)$ .
- At node  $S_2$ , compute  $\psi_D(d_1, s_1, d_2, v) = \int u_D(d_1, s_2, d_2, v) p_D(s_2 | s_1, d_2) ds_2$ .
- At node  $D_2$ , compute  $\psi_D(d_1, s_1, v) = \max_{\sum d_2^{ij} \leq D_2} \psi_D(d_1, s_1, d_2, v)$  and store optimal allocation.
- At node  $S_1$ , compute  $\psi_D(d_1, v, a) = \int \psi_D(d_1, s_1, v) p_D(s_1 | a, d_1) ds_1$ .
- At node  $A$ , compute  $\psi_D(d_1, v) = \int \psi_D(d_1, v, a) p_D(a | d_1) da$
- At node  $D$ , compute  $\psi_D(v) = \max_{\sum d_1^{ij} \leq D_1} \psi_D(d_1, v)$  and store optimal allocation.

$$\max_{\sum d_1^{ij} \leq D_1} \max_{\sum d_2^{ij} \leq D_2} \int \int \int u_D(d_1, s_2, d_2, v) p_D(s_2 | s_1, d_2) p_D(s_1 | a, d_1) p_D(a | d_1) ds_2 ds_1 da$$

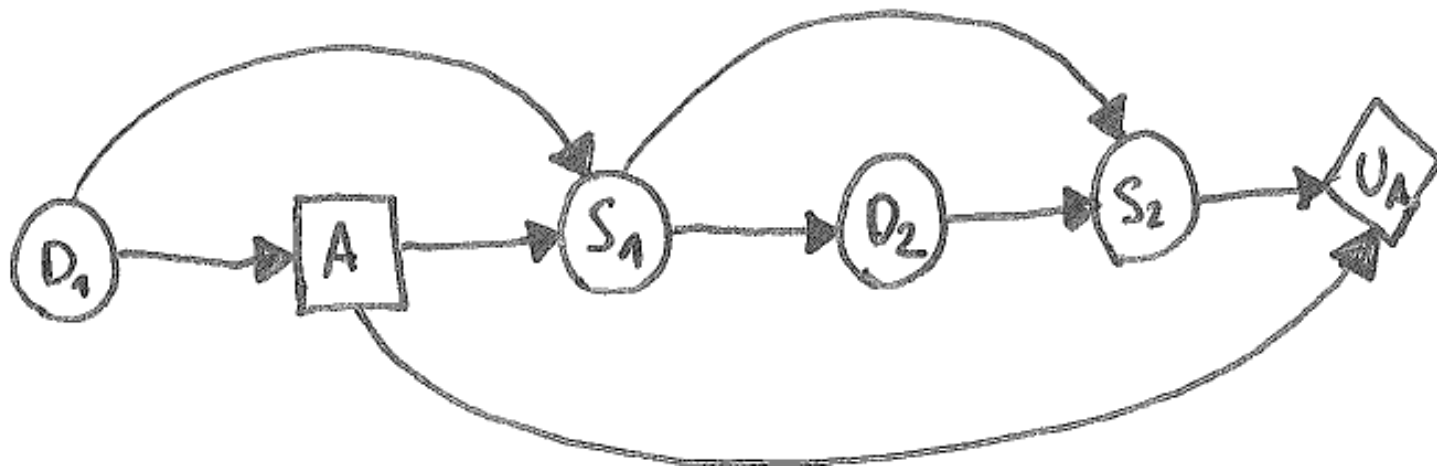
$$p_D(a | d_1)$$



# ARA for Urban Security. Mob dynamics

At each cell:

- Observes resource allocation  $d_{ij}^1$
- Undertakes attack  $a_{ij}$ , with impact  $s_{ij}^1$
- Observes recovery with resources  $d_{ij}^2$  with a level of success  $s_{ij}^2$
- Gets a consequence
- Aggregates utilities/Aggregates consequences

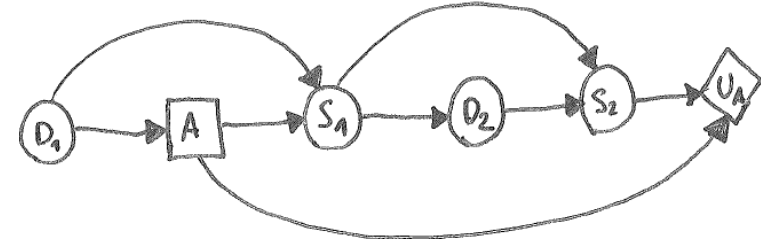


# ARA for Urban Security. Mob Dynamics

- The assessments for the Mob are
  - $p_A(d_2|s_1)$
  - $p_A(s_1|a, d_1)$
  - $p_A(s_2|s_1, d_2)$
  - $u_A(a, s_2, v)$
- We model our uncertainty about them through
  - 
  - $P_A(d_2|s_1)$
  - $P_A(s_1|a, d_1)$
  - $P_A(s_2|s_1, d_2)$
  - $U_A(a, s_2, v)$

# ARA for Urban Security. Mob dynamics

- We propagate such uncertainty through the scheme



- At node  $U_A$ ,  $U_A(a, s_2, v)$ .
- At node  $S_2$ , compute  $\Psi_D(a, s_1, d_2, v) = \int U_A(a, s_2, v) P_A(s_2|s_1, d_2) ds_2$ .
- At node  $D_2$ , compute  $\Psi_D(a, s_1, v) = \int \Psi_D(a, s_1, d_2, v) P_A(d_2|s_1) dd_2$
- At node  $S_1$ , compute  $\Psi_D(d_1, v, a) = \int \Psi_D(a, s_1, v) P_A(s_1|a, d_1) ds_1$ .
- At node  $A$ , compute  $\Psi_D(d_1, v) = \max_{\sum a^{ij} \leq A} \Psi_D(d_1, v, a)$  and stores optimal random allocation  $A^*(d_1, v)$ .

$$\int_{\{x \leq a\}} p_D(A = x | d_1, v) dx = Pr(A^*(d_1, v) \leq a)$$

# ARA for Urban Security. Mob dynamics

- We can estimate it by Monte Carlo

- Sample from

$$F = \{U_A(a, s_2, v), P_A(s_1 | a, d_1), P_A(d_2 | s_1), P_A(s_2 | s_1, d_2)\}$$

- Solve for maximum expected utility attack  
(EU computed in one step+ augmented prob.  
Simulation)

$$\hat{P}_D(A \leq a | d_1) = \frac{\#\{A_k^*(v, d_1) \leq a\}}{n}$$

# ARA for Urban Security. Mob dynamics

Inicializamos parámetros  $v, k, ro, D1, D2, A, \dots$

Generamos  $pA(d2|s1, a, d1)$

Para  $d1$ 's

Para  $i$ 's de 1 a  $n$

$c \leftarrow U(1, 10)$

Para  $a$ 's

Para  $s1$ 's

Generamos  $pA_i(s2_j|d2, s1)$

Nodo S2

$\psi_{A_i}(a, s1, d2, v) \leftarrow \sum_{s2} u_{A_i}(a, s2, v) * \prod_j pA_i(s2_j|d2, s1)$

Nodo D2

$\psi_{A_i}(d1, a, s1, v) \leftarrow \sum_{d2} \psi_{A_i}(a, s1, d2, v) * pA(d2|s1, a, d1)$

Nodo S1

Generamos  $pA_i(s1|a, d1)$

$\psi_{A_i}(d1, a, v) \leftarrow \sum_{s1} \psi_{A_i}(d1, a, s1, v) * \prod_j pA_i(s1_j|a, d1)$

Fin  $s1$ 's

Fin  $a$ 's

Nodo A

$a^*_i(d1, v) \leftarrow \operatorname{argmax}(\psi_{A_i}(a, d1, v))$

Actualizamos  $\{\# a^*_i(d1, v) = a\}$

Fin  $i$ 's

$pD^{\wedge}(a|d1) \leftarrow \{\# a^*_i(d1, v) = a\} / n$

Fin  $d1$ 's

# ARA for Urban Security. Mob dynamics

Very complex (De Werra's talk)

Parallelise

Do the computations at a few d1's and then some kind of response surface

Augmented simulation

Both problems in one shot??

# Discussion

- Computational scheme
- Higher k-level
- Multiple Defenders to be coordinated (risk sharing).
- Private security
- Multiple Attackers possibly coordinated
- Various types of resources
- Various types of delinquency
- Multivalued cells. The perception of security (concern analysis)
- Multiperiod planning
- Time and space effects (Displacement of delicts)
- Insurance
- General coupled influence diagrams
  
- Networks with value only at nodes
- Networks with value at nodes and arcs